Embedding Brans-Dicke gravity into electroweak theory

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We argue that a version of the four dimensional Brans-Dicke theory can be embedded in the standard flat spacetime electroweak theory. The embedding involves a change of variables that separates the isospin from the hypercharge in the electroweak theory.

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The construction of a consistent four dimensional quantum theory of gravity remains a challenge. Superstring theory with its pledge to unify all known interactions is the most attractive candidate for resolving this conundrum [1]. Some colleagues have also argued that conformal Weyl gravity is a renormalizable albeit not apparently unitary four dimensional quantum theory of gravity [2]. Finally, there are indications that $\mathcal{N}=8$ supergravity theory might be ultraviolet finite, for reasons that resemble those ensuring the finiteness of the $\mathcal{N}=4$ supersymmetric Yang-Mills theory [3]. The spectrum of the latter appears to coincide with that of the $AdS_5\times\mathbb{S}^5$ solution of ten dimensional IIB supergravity theory [4]. This duality has led [5] to propose that even within the strong and the electroweak components of the Standard Model there is an embedded quantum theory of gravity that remains to be discovered. (See also [6].)

In the present paper we inspect how a gravity theory could be embedded in the bosonic sector of the (Euclidean signature) Weinberg-Salam Lagrangian [7],

$$\mathcal{L}_{WS} = \frac{1}{4}\vec{F}_{\mu\nu}^{\,2} + \frac{1}{4}B_{\mu\nu}^2 + |D_\mu\phi|^2 + \lambda(\phi^\dagger\phi)^2 + \mu^2\phi^\dagger\phi \quad (1)$$

All our notations are exactly as in [7]. We decompose the Higgs field ϕ and the $SU_L(2)$ gauge field A_μ^a as follows,

$$\phi \;=\; \left(\sigma\, e^{i\alpha}\right)\cdot\mathcal{S}\,, \quad \mathcal{S} = \frac{e^{i\gamma}}{\sqrt{2(1-n_3)}}\begin{pmatrix} n_1 - in_2 \\ 1-n_3\end{pmatrix} \quad (2)$$

$$\widehat{A}_\mu \;=\; \left\{\left(\mathcal{W}_\mu^3 + \frac{2i}{g}\,\vec{m}^+\cdot\partial_\mu\vec{m}^-\right)\hat{n} \;-\; \frac{i}{2g}\left[\,\partial_\mu\hat{n}\,,\,\hat{n}\,\right]\right\} + \left\{\mathcal{W}_\mu^+\cdot\hat{m}^+ \;+\; \mathcal{W}_\mu^-\cdot\hat{m}^-\right\} \;\equiv\; \hat{\mathcal{A}}_\mu + \hat{\mathcal{X}}_\mu \quad (3)$$

Here $\hat{n}$ is the isospin projection operator

$$\hat{n} \;=\; \vec{n}\cdot\vec{\tau} \;=\; -\,\frac{\phi^\dagger\vec{\tau}\phi}{\phi^\dagger\phi}\cdot\vec{\tau}$$

and $\vec{m}^\pm = \vec{m}^1 \pm i\vec{m}^2$ so that $(\vec{m}^1,\vec{m}^2,\vec{n})$ is a right-handed orthonormal triplet. If the phases α and γ are combined, (2), (3) involves sixteen independent fields as a complete decomposition should. But for future reference we prefer to keep α and γ separate. Notice that $\vec{m}^\pm$ is defined up to a phase that we may identify with γ. We have selected (2) and (3) so that in the unitary gauge where $\hat{n}$ becomes equal to the diagonal Pauli matrix τ^3, the diagonal component of the gauge field coincides with $\mathcal{W}_\mu^3$ and the off-diagonal components coincide with the $\mathcal{W}_\mu^\pm$. In the low-temperature Higgs phase $\mathcal{W}_\mu^3$ then combines with the $U_Y(1)$ hypergauge field B_μ into the massive neutral Z-boson and the massless photon, and the off-diagonal gauge fields $\mathcal{W}_\mu^\pm = \frac{1}{\sqrt{2}}\left(\mathcal{W}_\mu^1 \mp i\mathcal{W}_\mu^2\right)$ become the massive charged W-bosons [7].

In the present paper we shall argue that a gravity theory emerges from (1) in terms of spin-charge decomposed variables. For this we start by noting that the Higgs field is a Lorentz scalar but with a nontrivial isospin and a nontrivial hypercharge. Thus we decompose it accordingly, and the result is displayed in (2). Since the physical Z-boson and photon are both charge neutral, there is no room for any spin-charge decomposition in $\mathcal{W}_\mu^3$ and B_μ. But the W-bosons are charged and they also have a nontrivial Lorentz spin, and following [8] we separate their spin from their charge by decomposing

$$\mathcal{W}_\mu^+ = \psi_1\mathbb{e}_\mu + \psi_2\bar{\mathbb{e}}_\mu \quad (4)$$

The $\psi_{1,2}$ are two complex scalars, they carry the charge of $\mathcal{W}_\mu^\pm$ [8]. The complex four-vector $\mathbb{e}_\mu$ carries spin, it is normalized according to

$$\mathbb{e}_\mu\mathbb{e}_\mu = 0 \qquad \& \qquad \mathbb{e}_\mu\bar{\mathbb{e}}_\mu = 1 \quad (5)$$

The decomposition (2) admits an internal $U_\phi(1)$ gauge symmetry: If we send $\alpha\to\alpha+\delta$ and $\gamma\to\gamma-\delta$ the Higgs field ϕ remains intact. Similarly, if we multiply ψ_1 and ψ_2 by a phase and $\mathbb{e}_\mu$ by the complex conjugate phase, (3) and (4) do not change under this internal $U_W(1)$ gauge transformation. The gauge fields for these internal symmetries are composite vector fields. For $U_\phi(1)$ we have $\Lambda_\mu = -i\mathcal{S}^\dagger\partial_\mu\mathcal{S}$ and for $U_W(1)$ we have $\mathcal{C}_\mu = i\bar{\mathbb{e}}\cdot\partial_\mu\mathbb{e}$.

We now define a number of auxiliary quantities. We start by introducing the three component unit vector

$$\vec{t} \ = \ \frac{1}{\rho^2} \cdot \left(\bar\psi_1 \ \ \bar\psi_2 \right) \vec\sigma \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

where $\rho^2 = |\psi_1|^2 + |\psi_2|^2$. With $g\mathcal{Y}_\mu = g\,\vec{n}\cdot\vec{\mathcal{A}}_\mu - 2\Lambda_\mu$ we define a $U_\phi(1)\times U_W(1)$ covariant derivative as follows,

$$\begin{aligned} \mathrm{D}^{\mathcal{C}}_\mu \psi_{1,2} \ &= \ (\partial_\mu + ig\mathcal{Y}_\mu \mp i\mathcal{C}_\mu)\psi_{1,2}\,, \\ \mathrm{D}^{\mathcal{C}}_\mu \mathbb{e}_\nu \ &= \ (\partial_\mu + i\mathcal{C}_\mu)\mathbb{e}_\nu \end{aligned}$$

and a $U_\phi(1)\times U_Y(1)$ covariant derivative as follows,

$$\mathfrak{D}_\mu = \partial_\mu - i\frac{g}{2}\mathcal{Y}_\mu - i\frac{g'}{2}B_\mu$$

We then introduce the gauge invariant supercurrents

$$\begin{aligned} J^\pm_\mu \ &= \ \frac{i}{2\rho^2}\{ (\bar\psi_1 \mathrm{D}^{\mathcal{C}}_\mu \psi_1 - \psi_1 \bar{\mathrm{D}}^{\mathcal{C}}_\mu \bar\psi_1) \pm (1\to 2)\} \\ T_\mu \ &= \ -\frac{i}{2\sigma^2}\{\phi^*\mathfrak{D}_\mu\phi - \phi\,\bar{\mathfrak{D}}_\mu\phi^*\} \end{aligned} \qquad (6)$$

Finally,

$$P_{\mu\nu} = \frac{1}{2} ig\cdot\rho^2 t_3 \cdot (\mathbb{e}_\mu \bar{\mathbb{e}}_\nu - \mathbb{e}_\nu \bar{\mathbb{e}}_\mu) \equiv g\rho^2 t_3 H_{\mu\nu}\,.$$

Following [8] we interpret ρ^2 as the conformal scale of a locally conformally flat metric tensor,

$$G_{\mu\nu} \ = \ \left(\frac{\rho}{\kappa}\right)^2 \delta_{\mu\nu} \qquad (7)$$

and from now on all the Greek indices $\mu,\nu,\lambda,...$ refer to the ensuing locally conformally flat spacetime. Note that in a coordinate basis the metric tensor is dimensionless while ρ has the dimensions of mass. Dimension analysis then tells us to introduce the *a priori* arbitrary mass parameter κ. With the metric tensor we have the vierbein $E^a{}_\mu$ that relates a coordinate basis (μ) to a local orthogonal frame (a), the Christoffel symbol $\Gamma^\lambda_{\mu\nu}$, the spin connection $\omega_\mu{}^\lambda{}_\nu$ and all other geometric quantities that are defined in the usual, standard fashion. The covariant derivative of the zweibein field $\mathbb{e}_\mu$ is [8]

$$\nabla_\mu \mathbb{e}_\nu + \omega_\mu{}^\lambda{}_\nu \mathbb{e}_\lambda = \partial_\mu \mathbb{e}_\nu - \Gamma^\lambda_{\mu\nu}\mathbb{e}_\lambda + \omega_\mu{}^\lambda{}_\nu \mathbb{e}_\lambda = \rho\cdot\partial_\mu(\frac{\mathbb{e}_\nu}{\rho})$$

The covariantized $U_W(1)$ gauge field is

$$\mathcal{C}_\mu = i\bar{\mathbb{e}}^\sigma \nabla_\mu \mathbb{e}_\sigma + i\bar{\mathbb{e}}^\lambda \omega_\mu{}^\sigma{}_\lambda \mathbb{e}_\sigma$$

and we also introduce the following twisted covariant derivative operator

$$\mathcal{D}_\mu{}^\nu{}_\lambda = \delta^\nu{}_\lambda \nabla^{\mathcal{C}}_\mu + \omega_\mu{}^\nu{}_\lambda = \delta^\nu{}_\lambda(\nabla_\mu + i\mathcal{C}_\mu) + \omega_\mu{}^\nu{}_\lambda$$

Finally, in $4D$ the Ricci scalar for our metric tensor is

$$R = -6\left(\frac{\kappa}{\rho}\right)^2 \left\{ \frac{1}{\rho^2}(\partial_\mu\rho)^2 \ + \partial_\mu(\frac{1}{\rho}\,\partial_\mu\rho) \right\}$$

In order to relate (1) to a gravity theory, we first employ the present geometrical structure to convert it into a generally covariant form. The result is a sum of two terms $\mathcal{L}_{WS} = \mathcal{L}^{(1)}_{WS} + \mathcal{L}^{(2)}_{WS}$ that we now inspect separately. We start with $\mathcal{L}^{(1)}_{WS}$. It admits two contributions, the first one is

$$\mathcal{L}^{(11)}_{WS} = \frac{1}{4}\sqrt{G}G^{\mu\nu}G^{\rho\sigma}\mathfrak{G}_{\mu\rho}\mathfrak{G}_{\nu\sigma} \qquad (8)$$

$$+\frac{1}{4}\sqrt{G}G^{\mu\nu}G^{\rho\sigma}\mathcal{T}_{\mu\rho}\mathcal{T}_{\nu\sigma} + \varsigma^2\sqrt{G}G^{\mu\nu}T_\mu T_\nu \qquad (9)$$

$$+\sqrt{G}G^{\mu\nu}\partial_\mu\varsigma\partial_\nu\varsigma + \frac{\varsigma^2}{6}\cdot R\sqrt{G} + \sqrt{G}\{\lambda\varsigma^4 + r\varsigma^2\} \qquad (10)$$

We have defined

$$\mathfrak{G}_{\mu\nu} = \mathcal{G}_{\mu\nu}(\mathcal{C}) - (\partial_\mu J^+_\nu - \partial_\nu J^+_\mu) - 2g^2\kappa^2 t_3 H_{\mu\nu} \qquad (11)$$

where $\mathcal{G}_{\mu\nu}(\mathcal{C})$ is the 't Hooft tensor [9]

$$\mathcal{G}_{\mu\nu}(\mathcal{C}) = \partial_\mu[t_3\mathcal{C}_\nu] - \partial_\nu[t_3\mathcal{C}_\mu] - \frac{1}{2}\vec{t}\cdot\partial_\mu\vec{t}\times\partial_\mu\vec{t} \qquad (12)$$

and

$$\mathcal{T}_{\mu\nu} = \frac{1}{g'}\left[\mathcal{G}_{\mu\nu}(\mathcal{C}) - (\partial_\mu J^+_\nu - \partial_\nu J^+_\mu) - 2(\partial_\mu T_\nu - \partial_\nu T_\mu)\right]$$

and $\varsigma = G^{-1/8}\,\sigma$ and $r = G^{-1/4}\mu^2$. Note in particular that (8)-(10) have no explicit κ dependence except for the last term in $\mathfrak{G}_{\mu\nu}$.

The second contribution to $\mathcal{L}^{(1)}_{WS}$ is

$$\mathcal{L}^{(12)}_{WS} = \kappa^2\cdot\left\{\frac{1}{2}\sqrt{G}\,G^{\mu\nu}\left[J^+_\mu J^+_\nu + \frac{1}{4}\nabla^{\mathcal{C}}_\mu\vec{t}\cdot\nabla^{\mathcal{C}}_\nu\vec{t}\right.\right. \qquad (13)$$

$$\left. +(\bar{\mathcal{D}}_\mu{}^\sigma{}_\lambda\bar{\mathbb{e}}_\sigma)(\mathcal{D}_\nu{}^\tau{}_\eta\mathbb{e}_\tau) + \frac{1}{2}t_-(\mathcal{D}_\mu{}^\sigma{}_\lambda\mathbb{e}_\sigma)(\mathcal{D}_\nu{}^\tau{}_\eta\mathbb{e}_\tau) + c.c.\right] \qquad (14)$$

$$\left. +\frac{1}{12}\cdot R\sqrt{G} + \frac{1}{4}g^2(\varsigma^2 - \frac{3}{8}t_3^2\kappa^2\}\cdot\sqrt{G}\right\} \qquad (15)$$

Notice that the entire (13)-(15) is proportional to κ^2.

Before we proceed to the final term

$$\mathcal{L}^{(2)}_{WS} \ = \ -\frac{1}{2}(D^{ab}_\mu[\mathcal{A}]\mathcal{X}_{\mu b})^2 \qquad (16)$$

we first point out some salient features in the structure of (8)-(10), (13)-(15):

We start by observing that $\mathcal{L}^{(1)}_{WS}$ involves only $SU_L(2)\times U_Y(1)$ gauge independent variables. There are sixteen independent fields, in addition to the $U_W(1)$ phase.

Paramount to our geometric interpretation of $\mathcal{L}^{(1)}_{WS}$ is that the density ρ is a nonvanishing quantity, $\langle\,\rho\,\rangle = \Delta \neq 0$. *A priori* it could be natural to identify $\Delta\equiv\kappa$ but we keep them separate. Arguments have been given [10], [11] that in a $SU(2)$ Yang-Mills theory Δ is nonvanishing. Assuming that this persists in the Weinberg-Salam

model, the Lagrangian (8)-(10), (13)-(15) is defined in a locally conformally flat spacetime which is different from the flat $\mathbb{R}^4$ of perturbation theory. This emergence of a novel spacetime is in line with the *no-go* theorem [12], [13] that forbids an embedded theory of gravity from residing in the same spacetime with the underlying nongravity theory; see also [5].

The contribution (15) has the standard Einstein-Hilbert form with a cosmological "constant" term. Similarly the two first terms in (10) constitute a Brans-Dicke Lagrangian: With $\psi = \varsigma^2$ we arrive at the standard Brans-Dicke form [14] with the conformally invariant parameter value $\omega = -\frac{3}{2}$.

The contribution (8) together with the first term in (13) describe the embedded dynamics of the supercurrent J_μ^+ with mass κ. The kinetic term of J_μ^+ is also embedded in the first term of (9). Similarly, the two terms in (9) describe the embedded dynamics of the supercurrent T_μ. It becomes massive whenever we are in a Higgs phase where ς acquires a nontrivial expectation value.

It is notable that when $\kappa \neq 0$ the vector J_μ^+ acquires a mass even in the absence of the conventional Higgs effect.

Both $\mathfrak{G}_{\mu\nu}$ and $\mathcal{T}_{\mu\nu}$ contain the 't Hooft tensor (12). Together with the kinetic term in (13) for the vector field $\vec{t}$ this gives us an embedded, unitary gauge version of the spontaneously broken $SO(3)$ Georgi-Glashow model. The unbroken symmetry group is the *compact* $U_W(1)$ that has the capacity of supporting embedded magnetic monopoles.

The second term in (12) in combination with the $\vec{t}$ contribution in (13) describes the embedding of the Faddeev model [15]. Consequently we expect embedded knotted solitons [16] to be present. Furthermore, in a Lorentz invariant ground state we must have $t_3 = \pm 1$ [8] and this prevents $\vec{t}$ from supporting any massless modes.

The contribution (14) and the last term in $\mathfrak{G}_{\mu\nu}$ defines a (gauged) Grassmannian nonlinear sigma-model. Its properties are detailed in [8], [17]. Together the unit vector $\vec{t}$ and the complex vector $\mathbb{e}_\mu$ describe a six dimensional internal space with the structure of $\mathbb{S}^2 \times \mathbb{S}^2 \times \mathbb{S}^2$.

The last term in (10) and the second term in (15) combine into a cosmological "constant" contribution. The original constant parameter μ^2 has become a spacetime dependent variable r. We can interpret it as a background scalar curvature and we can combine it with the middle term in (10). In addition, the Brans-Dicke-Higgs field ς has a mass term which is proportional to κ. This mass together with the scalar curvature R in (10) influence how symmetry becomes broken by the Higgs potential. In particular, there can be regions in the spacetime where the symmetry is broken while in other spacetime regions symmetry remains unbroken [18].

In the vicinity of the (Lorentz invariant [8]) $t_3 = \pm 1$ ground state and when the field variables are slowly varying, we may delete all derivative contributions to the Lagrangian (8)-(10), (13)-(15). We also assume that r describes the entire ground state scalar curvature $\langle R \rangle$. When we minimize the ensuing potential for ς in (15) and account for the $H_{\mu\nu}$ in the first term of (8), we conclude that the (classical level) cosmological constant becomes vanishingly small when the background scalar curvature $r \sim \langle R \rangle$ is

$$r \cdot \Delta \; = \; \mu^2 \approx - \left(\sqrt{\frac{\lambda}{2}} g + \frac{g^2}{4} \right) \Delta \tag{17}$$

This gives

$$\langle \varsigma^2 \rangle \approx \frac{1}{2\sqrt{2}} \frac{g}{\sqrt{\lambda}} \cdot \kappa^2 \tag{18}$$

Suppose now that (17), (18) hold and that we are near the BPS limit so that λ is vanishingly small, and that κ is finite (*e.g.* of the order of the electroweak scale). The cosmological constant then vanishes and the effective Planck's mass in the second term of (10) can become very large. The vector fields J_μ and $\vec{t}$ both have a mass which is of the order of the electroweak scale, but the vector field T_μ becomes very massive.

Finally, in the London limit where ρ is constant, $\mathcal{L}_{WS}^{(1)}$ describes the interactions between $J_\mu^\pm$, T_μ, $\vec{t}$, $\mathbb{e}_\mu$ and ς in a flat spacetime which is different from the flat $\mathbb{R}^4$ where (1) is defined.

We now proceed to the remaining contribution (16). Notably it is independently $SU_L(2) \times U_Y(1)$ gauge invariant. It describes the interactive dynamics between the Grassmannian vector field $\mathbb{e}_\mu$ and the two complex currents

$$\mathcal{J}_\mu^{(\pm)} \; = \; \frac{1}{4} \Gamma^\nu_{\nu\mu} + \frac{1}{2} \partial_\mu \ln(1 \pm t_3) - \frac{i}{t_3 \pm 1} \cdot (\, J_\mu^+ \pm J_\mu^- \,)$$

It also acquires a form which is generally covariant *w.r.t.* the metric tensor (7). In particular, in parallel with (13)-(15) the entire contribution (16) is proportional to κ^2.

Since $\mathcal{J}_\mu^{(\pm)}$ contains $\Gamma^\nu_{\nu\mu}$, the presence of (16) breaks the invariance under four dimensional diffeomorphism group *Diff(4)* into *SDiff(4)*, its volume preserving subgroup. But we have also observed that when $\kappa \neq 0$ both vector fields $J_\mu^\pm$ and T_μ are massive in the Higgs phase. As a consequence whenever $\kappa \neq 0$ the physical photon field becomes subject to the Meißner effect and acquires a mass in the Higgs phase. But since the photon mass (if there is any!) is tiny [19], in order for us to reconcile with the observed Physics we must take the limit $\kappa \to 0$. Since both (13)-(15) and (16) are proportional to κ^2 this truncates the entire Lagrangian into (8)-(10).

We are now in the position to state the main proposal of the present paper: *When the metric tensor* $G_{\mu\nu}$ *in the Lagrangian* (8)-(10) *is taken to be arbitrary, this Lagrangian describes the gravity theory that emerges from the electroweak Lagrangian* (1).

Since the 't Hooft tensor (12) is closed it can be written as the exterior derivative of a (generally singular)

vector field, and this vector field can be combined with J^+_μ. Thus both $\vec{t}$ and $\mathbb{e}_\mu$ entirely disappear from (8)-(10) when $\kappa \to 0$. When the metric tensor is arbitrary but *Diff(4)* symmetry remains broken into volume preserving *SDiff(4)* as $\kappa \to 0$, the Lagrangian (8)-(10) with a *a priori* arbitrary metric tensor engages sixteen *SDiff(4)* invariant fields and this coincides exactly with the number of $SU_L(2) \times U_Y(1)$ invariant fields in (1): The gravity Lagrangian describes the interactive dynamics of the conformal Brans-Dicke theory with a massless J^+_μ and with a T_μ that acquires a mass in the Higgs phase of the Brans-Dicke-Higgs scalar field ς^2.

The $\kappa \to 0$ limit is like a Wigner-Inönü contraction: The identification (7) becomes singular but at the level of the Lagrangian (8)-(10), (13)-(15), (16) with the *arbitrary* and in particular κ-independent metric, the $\kappa \to 0$ limit is well defined.

Finally, we propose the following interpretation for the appearance of a locally conformally flat metric tensor in the original Lagrangian (8)-(10): We view this Lagrangian as the short distance limit of a higher derivative (one loop) renormalizable gravity theory [2] with the additional term

$$\Delta\mathcal{L} \sim \frac{1}{4\gamma^2} \cdot W^2_{\mu\nu\rho\sigma}$$

where $W_{\mu\nu\rho\sigma}$ is the Weyl tensor. Since the coupling γ is asymptotically free, the β-function for γ enforces the Weyl tensor to vanish at the short distance limit [2]. This reduces the general metric tensor into its locally conformally flat form in the short distance limit; see also [8].

In conclusion, we have constructed a change of variables that converts the bosonic Weinberg-Salam Lagrangian into a variant of the Brans-Dicke Lagrangian in a locally conformally flat spacetime. We have argued that when the metric tensor becomes arbitrary and we take the limit where the physical photon becomes massless, this Brans-Dicke Lagrangian determines the gravity theory which is embedded in the Weinberg-Salam Lagrangian. We expect that one can similarly relate a gravity theory to the strong sector of the Standard Model. It would be interesting to work out the details in particular since the enlarged structure of the $SU(3)$ gauge group may directly engage the remaining components of a full metric tensor. We leave it as a puzzler to physically interpret the possibility that within the Standard Model there may be two distinct embedded gravity theories with their own distinct spacetimes.

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